(2) Einstein Theory (Quantum theory) :-

There is only one change, Einstein made in classical theory.

All the assumption of Einstein theory were same as classical theory But the only difference was ; now the Harmonic oscillators are Quantum Harmonic Oscillators.

& They are oscillating with quantised frequencies freq. is Quantised in terms of a Q. No. (n), & n can take values from $n = 0, 1, 2, 3 \cdots \infty$.

Energy of one-dim Quantum H.O. is

$$\boxed{E_n = \left(n + \frac{1}{2}\right)\hbar\omega} \quad , n = 0, 1, 2, 3 \cdots$$

At temp. t, Average energy of H.O. is

$$\boxed{\bar{E} = \frac{1}{2}\hbar\omega + \frac{\hbar\omega}{e^{\frac{\hbar\omega}{KT}} - 1}}$$

this $\frac{1}{2}\hbar\omega$ is not thermal.

This is the energy of 1 H.O., But Here N, 3D. H.O.

N, 3D H.O. is equavelent to $3N$, 1 Dim H.O.

So energy of $3N$, 1 Dim H.O. is

$$\boxed{\bar{E} = \frac{3N}{2}\hbar\omega + \frac{3N\hbar\omega}{e^{\frac{\hbar\omega}{k_BT}} - 1}}$$

specific heat, $C_V = \left(\frac{\partial \bar{E}}{\partial T}\right)$

$$C_V = -3N\hbar\omega \left(e^{\frac{\hbar\omega}{K_B T}}-1\right)^{-2} \cdot e^{\frac{\hbar\omega}{K_B T}} \cdot \frac{\hbar\omega}{K_B}\left(\frac{-1}{T^2}\right)$$

$$C_V = \frac{3N(\hbar\omega)^2}{K_B}\left(\frac{1}{T^2}\right) \frac{e^{\hbar\omega/K_B T}}{\left(e^{\hbar\omega/K_B T}-1\right)^2}$$

Now put, $\theta_E = \frac{\hbar\omega}{K_B}$ (Einstein temp.)

$$\boxed{C_V = 3R\left(\frac{\theta_E}{T}\right)^2 \frac{e^{\theta_E/T}}{\left(e^{\theta_E/T}-1\right)^2}}$$

Einstein temp θ_E is reference temp.

<u>Case 1</u> :- At high temp. $\hbar\omega << KT$

$$\Rightarrow \theta_E << T$$

$e^{\frac{\theta_E}{T}}$ is very-very small. By Binomial expansion,

$$C_V = 3R\left(\frac{\theta_E}{T}\right)^2 \frac{e^{\theta_E/T}}{\left(\cancel{1}+\frac{\theta_E}{T}-\cancel{1}\right)^2}$$ (neglect $e^{\theta_E/T}$)

$$\boxed{C_V = 3R}$$

This theory matches with classical theory at high temp. It gives correct results at high temp.

<u>Case-2</u> :- At low temp. $\hbar\omega >> KT$

$$\Rightarrow \theta_E >> T$$

$$C_V = 3R\left(\frac{\theta_E}{T}\right)^2 \frac{e^{\theta_E/T}}{e^{2\theta_E/T}}$$ (neglect 1)

$$\boxed{C_V = 3R\left(\frac{\theta_E}{T}\right)^2 e^{-\theta_E/T}}$$

Here exponential term i.e. $e^{-\theta_E/T}$ is dominating.